

# The Implication Of Nonlinear Effect On DS/CDMA With Imperfect Power Control In Optical Transmission

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## Abstract

The impact of imperfect power control on an asynchronous direct sequence code division multiple access (DS/CDMA) system in optical transmission is discussed. The performance of the system is evaluated using the derived bit-error-rate (BER) by taking into account the intermodulation distortion due to the laser diode nonlinearity.

## Introduction

RECENTLY, spread-spectrum (SS) CDMA application for third-generation mobile radio has enjoyed an increased interest [1, 2]. It is well known that CDMA has the advantages of the large user capacity, the effective utilization of the frequency and immunity to multipath fading over other multiple access techniques [3]. On the other hand, in the field of mobile radio applications, microcellular radio communication systems are expected to provide flexible telephony service for wireless personal communications. One of the key issue in implementing these systems is cost-effectiveness and flexible access capacity.

While most conventional wireless systems have been constructed on the conception that the radio wave is tried to be propagated as far as possible, conversely, future wireless systems will be required to have the ability to confine the radio wave within a small zone as in microcellular and picocellular systems, for enabling more channels and/or broadband services. In that sense, an optical-linked microcellular communication system with millimeter wave air interface would be promising hereafter [4]. In such a system, mobile stations communicate to the base stations (BSs) through a radio channel, whereas the BSs are connected to the control station (CS) through optical fiber. In DS/CDMA optical feeding system between BS and CS, the asynchronous SS signals are used to modulate

the optical intensity of the laser diode. However, the system suffers from intermodulation distortion caused by laser nonlinearities. This has been studied for the case when all mobile users generate equal powers in the receiver of their base station [5]. However, this is not accomplished perfectly in practical systems which results in capacity loss [6]. In this paper, the effect of imperfect power control on CDMA systems in optical transmission is studied taking into account the nonlinearity of the laser diode and its influence on the BER.

## CDMA System Model in Optical Transmission

The basic signal CDMA modulation model in optical transmission is shown in Figure 1. The asynchronous CDMA signals are assumed to be unequal due to the imperfection in the power control mechanism. In the following analysis, the effect of laser nonlinearity on these CDMA signals is discussed.

The  $k^{th}$  mobile user that transmits data continuously and asynchronously is presented by the SS signal  $s_k(t - \tau_k)$  ( $k = 1, 2, \dots, K$ ) as

$$\sqrt{2P_k} s_k(t - \tau_k) = \sqrt{2P_k} d_k(t - \tau_k) a_k(t - \tau_k) \cos(\omega_c t + \phi_k) \quad (1)$$

where  $\omega_c$  is the common center frequency,  $\phi_k$  ( $0 \leq \phi_k \leq 2\pi$ ) is the phase of the  $k^{th}$  carrier,  $\tau_k$  ( $0 \leq \tau_k \leq T$ ) is the time delay of the  $k^{th}$  user,  $P_k$  is the electrical power of the  $k^{th}$  user, and  $T$  is the bit duration. The bit data stream of the  $k^{th}$  user is given by:

$$d_k(t) = \sum_{\ell=-\infty}^{\infty} d_{\ell}^{(k)} p_T(t - \ell T); \quad d_{\ell}^{(k)} \in \{-1, 1\} \quad (2)$$

where  $d_{\ell}^{(k)}$  is a bit data value in the interval  $[\ell T, (\ell + 1)T]$ , and  $p_T(t) = 1$  for  $0 \leq t \leq T$  and 0 otherwise.

Each carrier is phase and amplitude coded by a waveform  $a_k(t)$  given by:

$$a_k(t) = \sum_{n=-\infty}^{\infty} a_n^{(k)} p_{T_c}(t - nT_c); a_n^{(k)} \in \{-1, 1\} \quad (3)$$

where  $a_n^{(k)}$  is a periodic binary sequence. Each bit is assumed to be encoded with  $N$  chips ( $T = NT_c$ ).

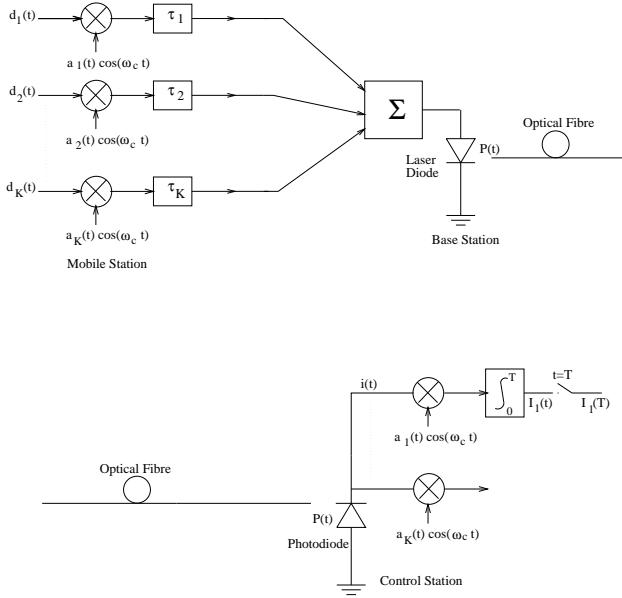


Figure 1: CDMA System Model in Optical Transmission

For the link between the base station and the control station, the sum of all the SS signals is used to modulate the optical intensity of the laser diode. The nonlinear characteristic of the laser diode produces intermodulation terms which influence the system performance. This memoryless nonlinearity is normally presented by a third-order polynomial [7] as

$$Y = A + P_t(X + A_2 X^2 + A_3 X^3) \quad (4)$$

where  $Y$  is the optical power output modulated by the current signal  $X$ , and  $A$ ,  $P_t$ ,  $A_2$ , and  $A_3$  are constants related to the characteristics of the laser diode used.

$$P(t) = P_t \left( 1 + m_0 \sum_{k=1}^K \sqrt{p_k} s_k(t - \tau_k) \right) \quad (5)$$

where  $P_t$  is the average transmitting optical power,  $m_0$  is the modulation index,  $p_k$  is a factor proportional to the  $k^{th}$  user power  $P_k$ , and  $P_k$  is assumed to be log-normally distributed according to [8],

$$f_{p_k}(p_k) = \frac{10 \log(e)}{\sqrt{2\pi} \sigma_P p_k} \exp\left(-\frac{(10 \log p_k)^2}{2 \sigma_p^2}\right) \quad (6)$$

where  $\sigma_P$  is the standard deviation of the  $k^{th}$  power. At the receiver, the optical intensity signal is converted to photo-current by a photodiode and then demodulated as shown in Figure 1. In addition to the laser diode relative intensity noise, shot noise and thermal circuit noise due to optical device, the multi-user interference limits the number of users and reduces the system performance.

The second-order intermodulation, caused by the square term of equation (4), generates zero frequency and double the signal frequency. The third-order intermodulation, caused by the presence of the cubic term in equation (4),  $\left(m_0 \sum_{k=1}^K \sqrt{p_k} s_k(t - \tau_k)\right)^3$ , generates the signal frequency and the triple signal frequency component. Therefore, in addition to the main signal component, only the third-order intermodulation is of interest as it is the only term that creates the interference components.

Without loss of generality, the signal from user 1 is assumed to be received. After demodulation at the receiver, and with some algebraic manipulations, the mean-square of the desired signal and the mean square of the interference can be derived in a similar way to [5] and are given respectively by:

$$I_0^2 = (\eta P_r)^2 (T m_0)^2 / 4 \int_0^\infty x^2 p(x) dx \quad (7)$$

and

$$\langle I^2 \rangle = (\eta P_r)^2 \sum_{m=1}^6 I_m^2 \quad (8)$$

where  $p(x)$  is the probability density function of  $\sqrt{P_k}$  and  $p(x)$  can be derived using  $p(x) = p(y)dy/dx$ .

$$I_1^2 = \frac{1}{2} (K-1) \left( \frac{3 A_3 m_0^3 T}{8} \right)^2 \int_0^\infty x^2 p(x) dx \int_0^\infty y^2 p(y) dy \quad (9)$$

here  $y = x^2$  and  $p(y)$  is the probability density function of  $P_k$  that is given by equation (6)

$$\begin{aligned} I_2^2 = & \frac{m_0^2 (K-1) T^2}{12 N} \left( \int_0^\infty x^2 p(x) dx + \left( \frac{3 A_3 m_0^2}{4} \right)^2 \right. \\ & \int_0^\infty z^2 p(z) dz + \int_0^\infty y^2 p(y) dy \\ & \left. \left( \frac{3 A_3 m_0^2}{2} + K(K-1) \frac{9 A_3^2 m_0^4}{4} \int_0^\infty x^2 p(x) dx \right) \right. \\ & \left. + 3(K-1) A_3 m_0^2 \left( \int_0^\infty x^2 p(x) dx \right)^2 \right) \quad (10) \end{aligned}$$

here  $z = x^3$  and  $p(z)$  can be derived using  $p(z) = p(x)dx/dz$ .

$$I_3^2 = \frac{3A_3^2 m_0^6 (K-1) T^2}{64N} \int_0^\infty x^2 p(x) dx \int_0^\infty y^2 p(y) dy \quad (11)$$

$$I_4^2 = \frac{3A_3^2 m_0^6 (K-1) (K-2) T^2}{64N} \int_0^\infty x^2 p(x) dx \int_0^\infty y^2 p(y) dy \quad (12)$$

$$I_5^2 = \left( \frac{A_3 m_0^3}{4} \right)^2 \frac{15T^2}{4N} (K-1)(K-2) \left( \int_0^\infty x^2 p(x) dx \right)^3 \quad (13)$$

and

$$I_6^2 = \left( \frac{A_3 m_0^3}{4} \right)^2 \frac{3T^2}{20N} (K-1)(K-2) (K-3) \left( \int_0^\infty x^2 p(x) dx \right)^3 \quad (14)$$

For a given bandwidth  $B$ , the BER is given by  $BER = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{SNR}{2}}$ , where the signal-to-noise ratio is given by

$$SNR = \frac{NI_0^2}{BT^2(\langle I_{LD}^2 \rangle + \langle I_{shot}^2 \rangle + \langle I_{th}^2 \rangle)} + \frac{I_0^2}{\sum_{m=1}^6 I_m^2} \quad (15)$$

$\langle I_{LD}^2 \rangle = RIN(\eta P_r)^2$  is the noise due to the fluctuation in the laser diode output intensity,  $\langle I_{shot}^2 \rangle = 2enP_r$  is the shot noise at the receiver, and  $\langle I_{th}^2 \rangle$  is the thermal circuit noise. The bandwidth  $B$  is assumed to be 30 N (kHz) [5].

## Results

The power control mechanism for CDMA systems has been tested via simulation and has shown that received signal power has a log-normal distribution with a standard deviation in the vicinity of 1 dB above the desired power [9]. Hence, results will be generated for  $\sigma_P = 1$  dB as the worst case. Using the derived equations, a system with  $N = 127$  has been selected. A  $1.3 \mu\text{m}$  laser diode with a RIN intensity noise of -150 dB/Hz and output power of 2 mW has been chosen. The photodiode sensitivity is  $\eta = 0.8mA/mW$  and

the photodiode thermal noise is  $I_{th} = 5pA/\sqrt{Hz}$ . Figure 2 shows the BER versus the modulation index  $m_0$  for an optical power of -40 dBm for selected number of users,  $K$ , for the case with ideal power control contrasted to the practical case with imperfect power control ( $\sigma_P = 1$  dB).

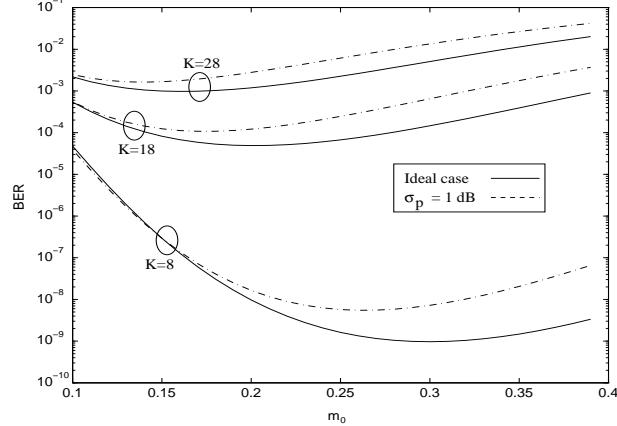


Figure 2: Bit-Error-Rate versus the modulation index  $m_0$  for different number of users  $K$  for the ideal case with perfect power control and the practical case with imperfect power control

For the  $K = 8$  case, it can be seen that the BER is degraded when  $m_0$  becomes greater than 0.15 whereas for the other cases it is degraded for values of  $m_0$  greater than 0.1. This is due to the noise domination in the region of low  $m_0$ . The interference due to the intermodulation terms is substantial at higher values of  $m_0$ .

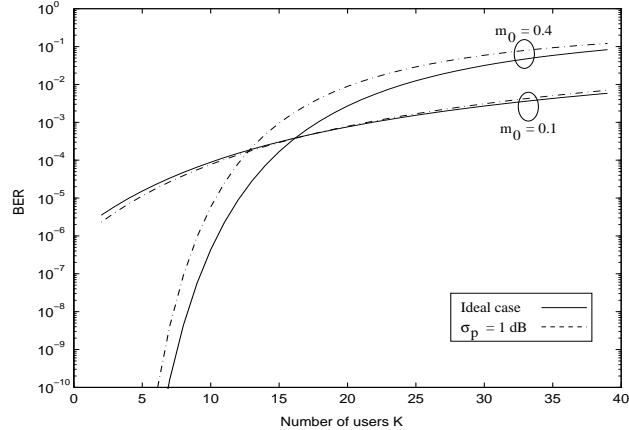


Figure 3: Bit-Error-Rate versus the number of users  $K$  for different values of the modulation index  $m_0$  for the ideal case with perfect power control and the practical case with imperfect power control

Figure 3 shows the BER versus the number of users,  $K$ , for selected values of the modulation index  $m_0$  for both the ideal case and the case with imperfect power control.

It can be easily seen from this figure that when the value of the modulation index is small, e.g.  $m_0 = 0.1$ , the effect of imperfect power control can yield better results when the number of users is small ( $K < 13$ ). The results displayed in Figures 2 and 3 therefore form the basis for the design of CDMA systems in optical transmission.

## Conclusion

The effect of the intermodulation in DS/CDMA systems with imperfect power control in optical transmission has been presented. A memoryless third-order polynomial was used to present the laser diode nonlinearity. The results achieved are useful for system design and performance analysis.

## Acknowledgement

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